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Numerical investigation of a simple model of human jumping on an oscillating structure

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Abstract

Cantilever grandstands are susceptible to vertical vibrations due to jumping or bobbing (vertical motion without loss of contact) of human occupants. Several major sports stadia have had vibration problems as a result. For bobbing, a simple mass-spring-damper-actuator model of the human (or group of humans) has been proposed and has been incorporated in the Institution of Structural Engineers guidelines on the dynamic performance of grandstands. For jumping, laboratory experiments have shown that in certain conditions regular periodic jumping close to the natural frequency of the supporting structure is not possible. However, there has previously been no model of the human-structure interaction in this situation. A fundamental difference from the bobbing case is that there are discontinuities between contact and non-contact phases of the motion. The behaviour is therefore highly nonlinear. This paper proposes a model similar to the human bobbing model but allowing for the loss of contact during jumping and the condition for landing, which is clearly influenced by the motion of the structure during the no-contact phase. The behaviour of the model is explored numerically. It is shown that in certain parameter ranges there are stable periodic solutions, but in others the periodic motion is unstable and instead the system exhibits chaotic motion. This could possibly explain the physical difficulty of regular periodic jumping close to the natural frequency of the supporting structure in the previous laboratory experiments.

Keywords: Human-structure interaction; Stadium vibrations; Human jumping loading; Nonlinear dynamics; Piecewise smooth dynamical system; Bifurcation to chaos

1. Introduction

Human-structure-interaction (HSI) has a significant effect on the loading and response of structural systems. This is modelled approximately by a linear spring-mass-damper-actuator system[1,2] that is incorporated in the IStructE guidelines [3]. This "bobbing" model maintains full contact between the person/crowd and structure. For the jumping case [4] showed, experimentally, that periodic jumping at frequencies close to the natural frequency of the structure was not possible. This behaviour cannot be explained by the simple linear [1,2] model. In this paper we present a low-order model for bobbing and jumping that includes both contact and flight phases. This non-smooth system exhibits a range of esoteric nonlinear responses that offer an explanation for experimental observations of [4].

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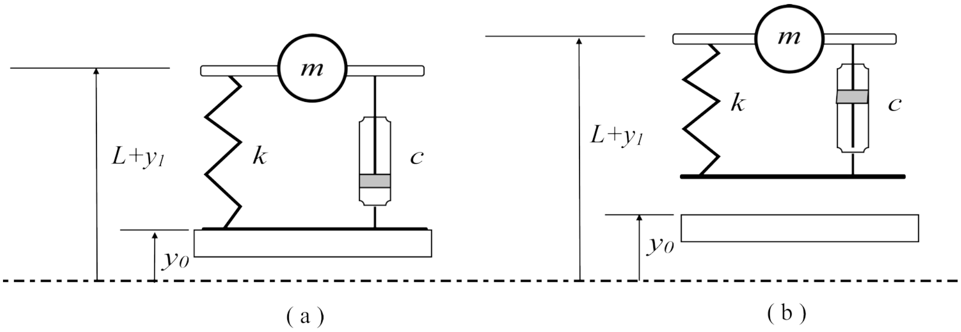

Fig. 1. Idealised model of passive jumper on moving structure (a) contact phase (b) flight phase

2. Theory

The system Lagrangian Π (kinetic minus potential energy) and a Rayleigh dissipative function R for the model shown in fig. 1 with no loss of contact is as follows:

$$\Pi = \tfrac{1}{2}m\dot{y}_0^2 - \left\{\tfrac{1}{2}k\left(y_1 - y_0\right)^2 + mg\int \mathrm{d}\,(y_1 + L) - \int F\mathrm{d}y_0\right\}, \qquad \mathrm{R} = \tfrac{1}{2}c(\dot{y}_1 - \dot{y}_0)^2 \tag{1}$$

where m is the person's mass, k is the person's (passive) stiffness, L is the person's unloaded "leg length" and F is the force of the person on the structure and g is the gravitational acceleration constant. The ground vertical absolute position of the structure (time-varying) is given by y_0 and the absolute position of the person's centre of mass is $L + y_1$. Introducing the following system parameters,

$$\omega^2 = k/m, \quad 2\gamma\omega = c/m, \tag{2}$$

where ω is the passive natural circular frequency of the person and γ is the ratio of critical damping. The equations of motion can be completely non-dimensionalised by scaling time and length as follows

$$t = \tau/\omega, \quad F = mgf, \quad y_s = -g\big/\omega^2, \quad y_1 = |y_s|\,u_1, \quad y_0 = |y_s|\,u_0, \tag{3}$$

where y_s is the static displacement of mass m due to its self weight. The contact force ratio f is the contact force divided by the static weight mg of the idealised jumper. Hence the equation of motion is defined as follows:

$$\ddot{u}_1 = -1 - f, \quad f = \begin{cases} 2\gamma\,(\dot{u}_1 - \dot{u}_0) + (u_1 - u_0) : & f \le 0 \quad \text{(contact phase)} \\ 0 & : \textit{otherwise} \quad \text{(flight phase)} \end{cases} \tag{4}$$

where f is the total contact force (non-dimensional) between the person and structure. Let the structural displacement u_0 be defined as

$$u_0 = \eta\cos(\Omega\tau), \quad \Omega = \omega_f\big/\omega, \quad T = 2\pi/\Omega \tag{5}$$

where ω_f is the dimensional oscillating frequency of the structure (the system's excitation frequency), η is the non-dimensional amplitude of the structure's oscillations and Ω is the non-dimensional excitation frequency ratio (excitation to person's natural frequency). A Poincaré section is defined as a stroboscopic sampling (of solutions) in scaled time at a period T which is defined with respect to the forcing frequency ratio. Poincaré points lie in the Poincaré section. Thus variable τ/T used in later figures indicates the number of forcing cycles.

2.1. Physical admissibility of generalised coordinates

There is no clear upper limit on deflection of the body mass in the case presented, given that the input energy of the structure is unknown. However, the minimum relative displacement of the body mass with respect to the structure

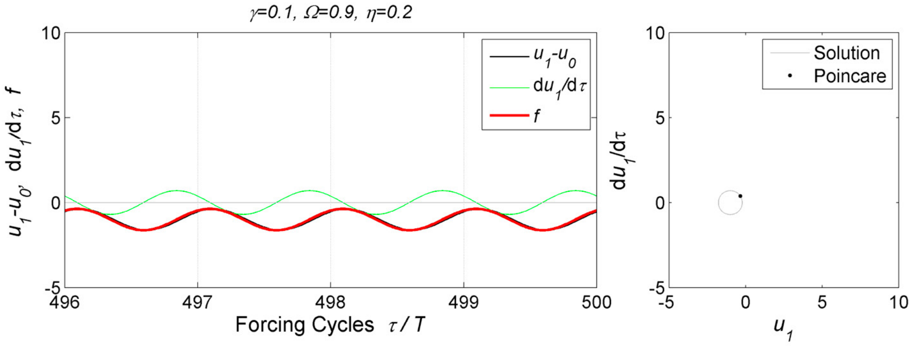

Fig. 2. Example of period 1 bobbling solution (no flight phase)

must be positive, i.e. $L+y_1-y_0 \geq \beta L$. Additionally we limit β to a range of geometrically linear biomechanics. Various empirical models for vertical stiffness of a person during running and jumping exist in the literature [5]. Here we chose the simplest model after [6] where $k = m\omega_j^2$ where m is the person's mass and ω_j is the jumping frequency (frequency of oscillation). In this paper we are interested in jumping at or around the person's natural frequency. Hence it is assumed that $\omega_j = \omega$ and therefore the vertical stiffness k is assumed constant. The estimated natural frequency of a person with bent knees is 3Hz [7,9]. Therefore y_s is assumed approximately 2.8cm and this corresponds to about $0.017L$. Hence, a physical admissible limit on relative displacement can be estimated as follows,

$$u_1 - u_0 \geq (\beta - 1)/\alpha \approx (0.8 - 1)/0.023 = -12.8 \quad (6)$$

The maximum contact force that the human body can sustain is also limited. Ref [8] suggests the following approximate maximum peak force, without knee locking, of $5mg$ thus,

$$f = 2\gamma\,(\dot{u}_1 - \dot{u}_0) + (u_1 - u_0) \geq -5 \quad (7)$$

A stationary point in $u_1 - u_0$ occurs when $\dot{u}_1 - \dot{u}_0 = 0$. This suggests that the maximum contact force constraint, equation (7), governs rather than equation (6).

3. Numerical simulations and results

Equations (4) are a set of piece-wise linear equations of motion. This non-smooth system (here piece-wise linear) results in a nonlinear system overall. Numerical solutions of equations (4) are obtained using Matlab's ode45 solver with event detection functions for determining the precise touch-down and take-off times. This system (4) and (5) becomes less stiff in free flight because for this part of a cycle the spring does no work. Hence it exhibits a softening nonlinear behaviour with increasing amplitude.

As a first exploration we consider a frequency parameter slightly below its linear small amplitude natural frequency, e.g. at $\Omega = 0.9$. Solutions of equations of motion (4), for various different structural amplitude η, exhibit several different periodicities, such as bobbing (Fig 2), low amplitude jumping (Fig 3), higher amplitude period 2 jumping (Fig 4) and high amplitude chaotic jumping (Fig 5). In Fig 5 the highest jump is $\max(u_1-u_0) = 10.8$ which corresponds (using equation (3)) dimensionally to a jump of approximately 30.2cm. Fig 3 demonstrates a technical issue with this simple model. Consider the lift-off case where the negative contact force increases to zero. At this point the spring is marginally compressed as $u_1 - u_0 < 0$ the difference is taken up by the out of phase damper force $2\gamma\,(\dot{u}_1 - \dot{u}_0)$. This is physically admissible. However, following the timeseries in Fig 3 to the next touch-down condition. At this point we have $f = 0$ and $u_1 - u_0 > 0$. The zero contact force event occurs at a point when the idealized jumper is technically in free-flight (for a very short period of time). This issue cannot be successfully treated without introduction of an additional degree of freedom for the foot. In this paper we shall assume that the foot has some compliance (due the ankle joint) which allows for this small necessary extension which is physically reasonable.

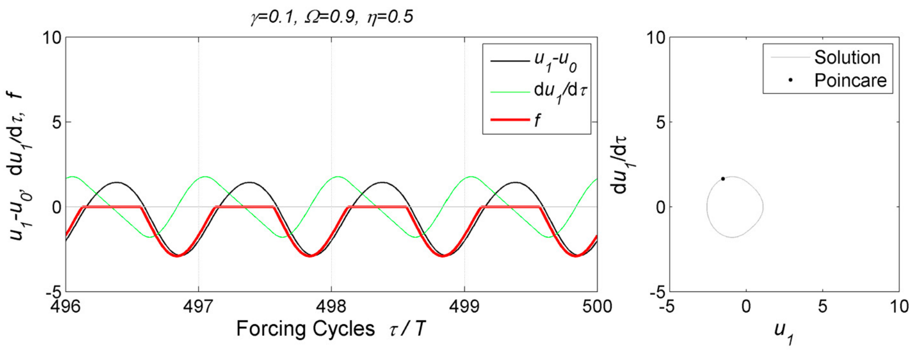

Fig. 3. Example of low amplitude period 1 jumping solution

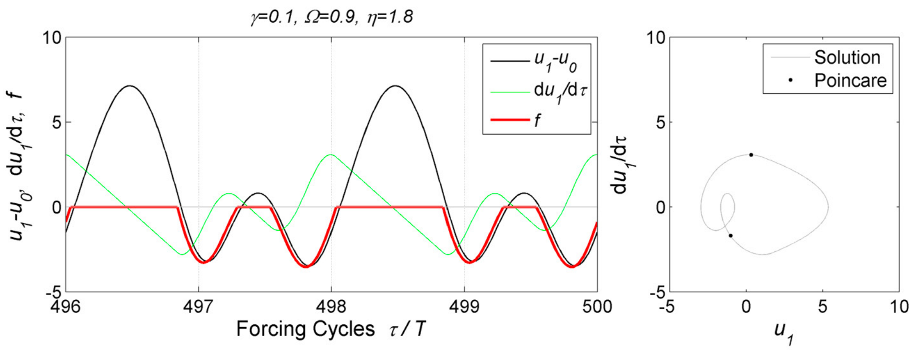

Fig. 4. Example of higher amplitude period 2 jumping solution

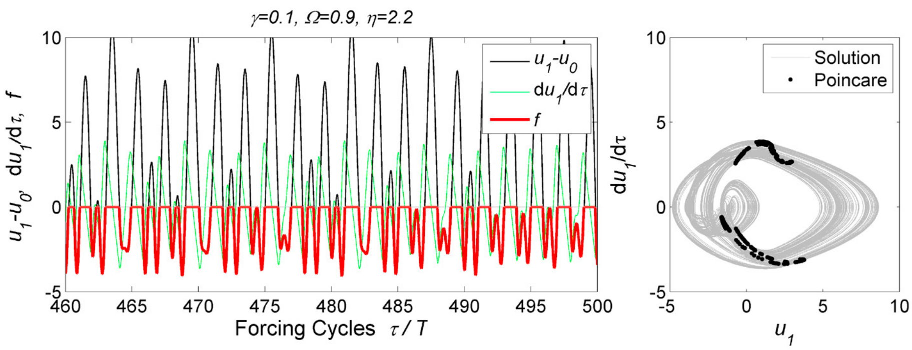

Fig. 5. Example of high amplitude chaotic jumping solution

Fig 6, is an amplitude sweep depicting the solution Poincaré points, and maximum and minimum contact forces, for increasing amplitude η. The first $200T$ of the solutions are discarded and so the transients (homogenous solutions) are attenuated. The initial conditions for $\eta + \delta\eta$ are assumed to be a Poincaré point of the solution at amplitude η. A grazing bifurcation between bobbing and jumping is observed at $\eta = 0.325$ and a period doubling bifurcation at $\eta = 1.58$ leading to a Feigenbaum cascade of period doubling to chaos at $\eta = 2.02$. Period 6 solutions are observed at $\eta = 2.18$ and period 4 at $\eta = 2.3$. At $\eta = 2.41$ a sudden explosion in magnitude of the chaos is observed, see Fig 7. This suggests the presence of coexisting chaotic attractors that need further exploration. At this level of forcing amplitude we exceed the physical admissibility limits (7). A reverse Feigenbaum cascade of period halving is seen and another sudden explosion of chaos at increasing amplitude. Figs 6, 8 and 9 are forcing amplitude sweeps around the linear resonance. Fig 8 at the linear resonance and fig 9 above the linear resonance doesnt exhibit the full Feigenbaum

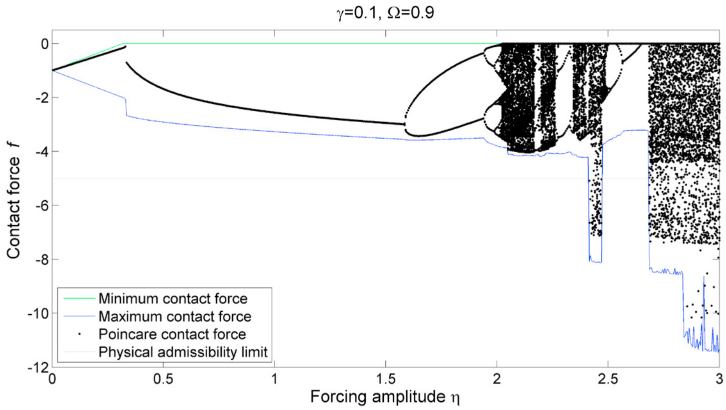

Fig. 6. Forcing amplitude sweep

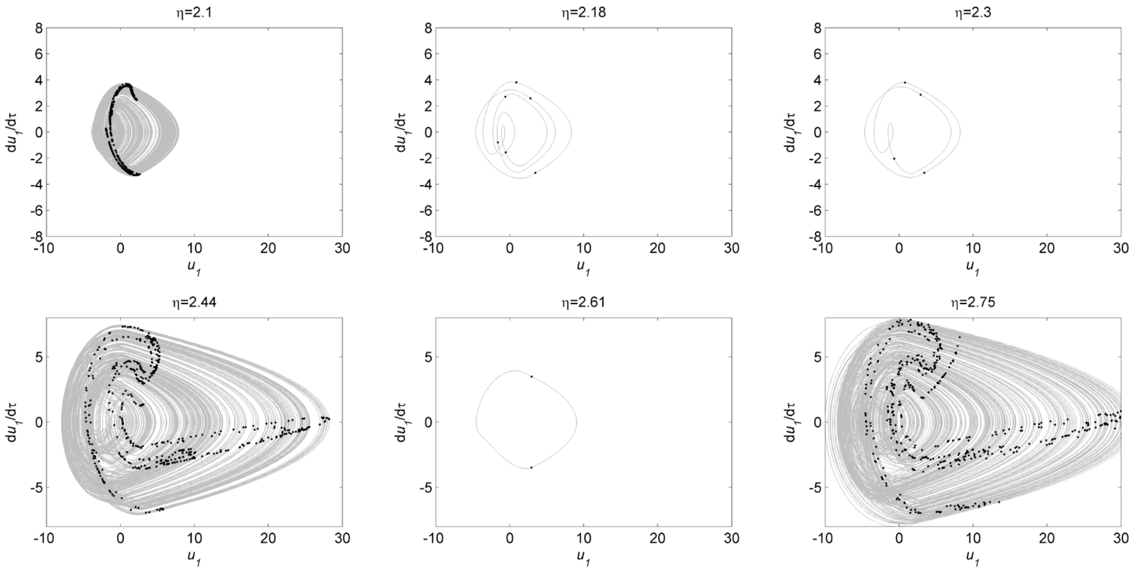

Fig. 7. Examples of high amplitude attractors ($\gamma = 0.1, \Omega = 0.9$)

cascade (within the physical admissibility limits) however at higher amplitudes the sudden chaotic explosion is still observed.

4. Conclusions

The results in this paper indicate that jumping at and around the linear natural frequency of a structure is possible for low amplitudes of jumping. At higher amplitudes of jumping period doubling cascades can be encountered. At a forcing frequency lower than the linear resonance a complete Feigenbaum cascade leading to chaotic jumping is observed. At forcing frequencies equal to and higher than the linear resonance we observe incomplete Feigenbaum cascades. This simple system exhibits a very diverse range of nonlinear dynamics. In conclusion this low-order model indicates that jumping at high amplitude around the linear resonant frequency is problematic for a person. This is because no stable period one jumping response attractors exist; only chaotic and high period stable responses are possible. Therefore this may be the explanation for experimental observations [4] - people cannot jump periodically at the same frequency as a structure's natural frequency.

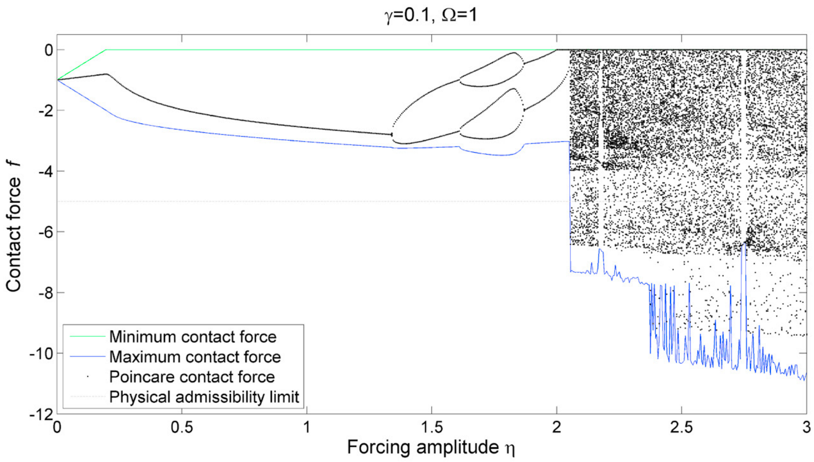

Fig. 8. Forcing amplitude sweep

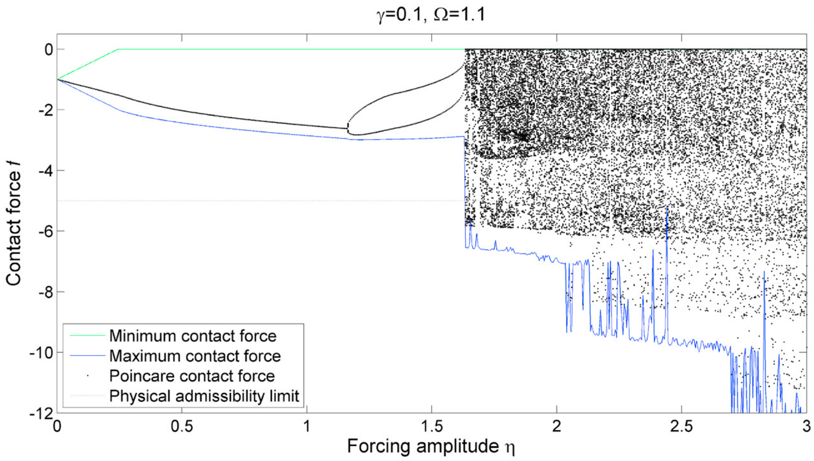

Fig. 9. Forcing amplitude sweep

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